Skill Builder: Topics 2.7 – Differentiating sin(x), cos(x), e^x, ln(x)

Find the derivative of each function.

1.) $f(\theta)=\frac{\pi}{2}\sin\theta-\cos\theta$ $f'(\theta)=\frac{\pi}{2}\cos\theta-\left(-\sin\theta\right)=\frac{\pi}{2}\cos\theta+\sin\theta$	**2.)** $y=x^2-\frac{1}{2}\cos x$ $\frac{dy}{dx}=2x-\frac{1}{2}\left(-\sin x\right)=2x+\frac{1}{2}\sin x$
3.) $f(x)=\frac{1}{2}e^x-3\sin x$ $f'(x)=\frac{1}{2}e^x-3\cos x$	**4.)** $f(x)=\frac{1}{x^2}-2e^x$ $f(x)=x^{-2}-2e^x$ $f'(x)=-2x^{-3}-2e^x=-\frac{2}{x^3}-2e^x$

Find the slope of the graph of the function at the given point. Use proper notation.

5.) $f(\theta)=4\sin\theta-\theta,\ (0,0)$ $f'(\theta)=4\cos\theta-1$ $f'(0)=4\cos 0-1=4-1=3$	**6.)** $f(x)=\frac{3}{4}e^x,\ \left(0,\frac{3}{4}\right)$ $f'(x)=\frac{3}{4}e^x \Rightarrow f'(0)=\frac{3}{4}$

Write the equation of the tangent line to the graph of the function at the given point

7.) $f(x)=x+e^x,\ (0,1)$ $f(0)=0+e^0=1$ $f'(x)=1+e^x \Rightarrow f'(0)=1+e^0=2$ tangent line : $y=1+2x$	**8.)** $g(t)=\sin t+\frac{1}{2}e^t,\ \left(\pi,\frac{1}{2}e^{\pi}\right)$ $g'(t)=\cos t+\frac{1}{2}e^t$ $g'(\pi)=\cos\pi+\frac{1}{2}e^{\pi}=-1+\frac{1}{2}e^{\pi}$ tangent line : $y=\frac{1}{2}e^{\pi}+\left(-1+\frac{1}{2}e^{\pi}\right)(x-\pi)$

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Determine the point(s) (if any) at which the graph of the function has a horizontal tangent line.

9.) $f(x) = -4x + e^x$ $f'(x) = -4 + e^x = 0 \Rightarrow e^x = 4 \Rightarrow x = \ln 4$ $f(\ln 4) = -4\ln 4 + 4$ horizontal tangent line at $(\ln 4, -4\ln 4 + 4)$	**10.)** $g(x) = x + \sin x,\ 0 \le x < 2\pi$ $g'(x) = 1 + \cos x = 0 \Rightarrow \cos x = -1 \Rightarrow x = \pi$ $g(\pi) = \pi + \sin \pi = \pi$ horizontal tangent line at (π, π)

For problems 11-14, use proper notation throughout.

11.) Consider the function $f(t) = \ln t$.

a.) Calculate the instantaneous rate of change of the function at $t = \frac{1}{2}$. $f'(t) = \frac{1}{t} \Rightarrow f'\left(\frac{1}{2}\right) = 2$	**b.)** Find the equation of the tangent line at the point where $t = 3$. Leave your answer in terms of the natural logarithm. $f'(3) = \frac{1}{3}$ $\quad f(3) - \ln 3$ tangent line : $y = \ln 3 + \frac{1}{3}(x - 3)$

Recall the various versions of the definition of derivative from Topics 2.1-2.3:

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}, \quad f'(c) = \lim_{h \to 0} \frac{f(c+h) - f(c)}{h} \text{ and } f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

12.) Find the following using the Limit Definition of Derivative. You should be able to do this with very little computation.

a.) $\lim\limits_{\Delta x \to 0} \frac{\sin(x + \Delta x) - \sin(x)}{\Delta x} = \frac{d}{dx}\sin x = \cos x$	**b.)** $\lim\limits_{h \to 0} \frac{(x+h)^2 - x^2}{h} = \frac{d}{dx}x^2 = 2x$
c.) $\lim\limits_{x \to \frac{\pi}{4}} \frac{\sin x - \sin\frac{\pi}{4}}{x - \frac{\pi}{4}} = \left.\frac{d}{dx}\right\|_{x=\frac{\pi}{4}} \sin x = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$	**d.)** $\lim\limits_{h \to 0} \frac{\sin\left(\frac{\pi}{6} + h\right) - \frac{1}{2}}{h} = \left.\frac{d}{dx}\right\|_{x=\frac{\pi}{6}} \sin x = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$
e.) $\lim\limits_{\Delta x \to 0} \frac{(2 + \Delta x)^3 - 8}{\Delta x} = \left.\frac{d}{dx}\right\|_{x=2} x^3 = 3(2)^2 = 12$	**f.)** $\lim\limits_{x \to \pi} \frac{\cos x + 1}{x - \pi} = \left.\frac{d}{dx}\right\|_{x=\pi} \cos x = -\sin\pi = 0$
g.) $\lim\limits_{x \to 2} \frac{\ln x - \ln 2}{x - 2} = \left.\frac{d}{dx}\right\|_{x=2} \ln x = \frac{1}{2}$	**h.)** $\lim\limits_{\Delta x \to 0} \frac{(3 + \Delta x)^2 + (3 + \Delta x) - 12}{\Delta x}$ $= \left.\frac{d}{dx}\right\|_{x=3} (x^2 + x) = 2(3) + 1 = 7$